Inference for Difference Between Many Proportions

Dr. Ab Mosca (they/them)

Slides based off slides courtesy of OpenIntro and John McGreedy of Johns Hopkins University

Plan for Today

- Chi-Square Test of Independence
- Goodness of Fit

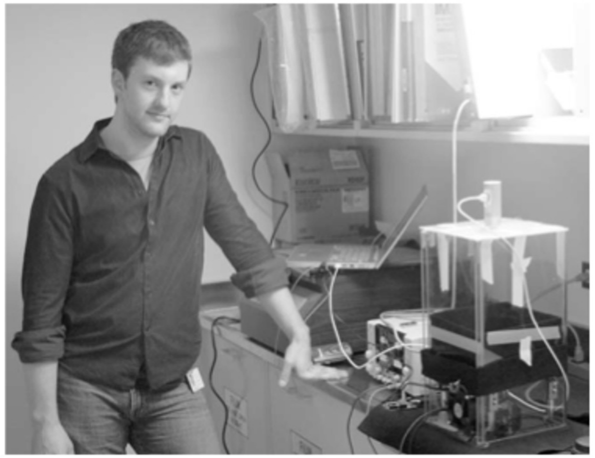
Chi-Square test of Goodness of Fit (GOF)

Weldon's dice

- Walter Frank Raphael Weldon (1860 - 1906), was an English evolutionary biologist and a founder of biometry. He was the joint founding editor of Biometrika, with Francis Galton and Karl Pearson.
- In 1894, he rolled 12 dice 26,306 times, and recorded the number of 5s or 6s (which he considered to be a success).

- It was observed that 5s or 6s occurred more often than expected, and Pearson hypothesized that this was probably due to the construction of the dice. Most inexpensive dice have hollowed-out pips, and since opposite sides add to 7, the face with 6 pips is lighter than its opposing face, which has only 1 pip.

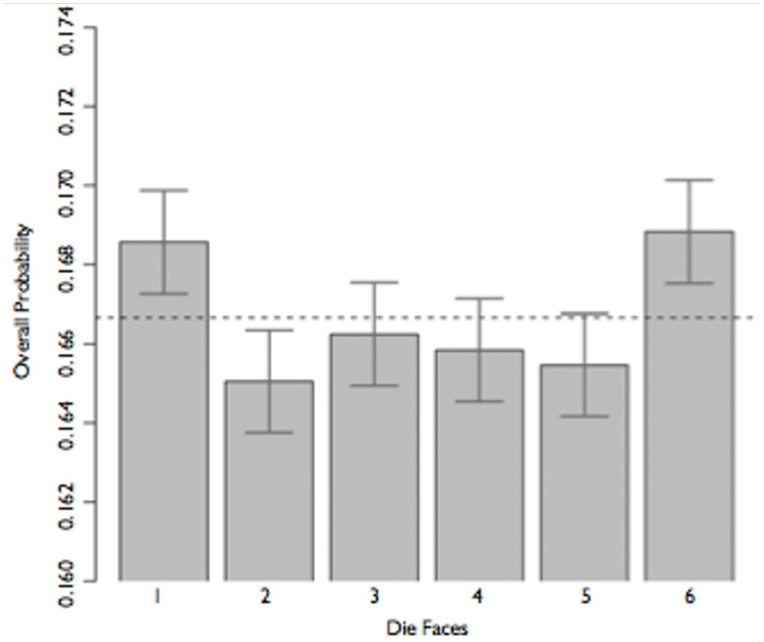
Labby's dice



- In 2009, Zacariah Labby (U of Chicago), repeated Weldon's experiment using a homemade dice-throwing, pip counting machine. www.youtube.com/watch?v=95EErdouO2w
- The rolling-imaging process took about 20 seconds per roll.
- Each day there were ~150 images to process manually.
- At this rate Weldon's experiment was repeated in a little more than six full days.

Labby's dice (cont.)

- Labby did not actually observe the same phenomenon that Weldon observed (higher frequency of 5s and 6s).
- Automation allowed Labby to collect more data than Weldon did in 1894, instead of recording "successes" and "failures", Labby recorded the individual number of pips on each die.



Expected counts

Labby rolled 12 dice 26,306 times. If each side is equally likely to come up, how many 1s, 2s, ..., 6s would he expect to have observed?

(a) 1/6
(b) 12/6
(c) 26,306 / 6
(d) 12 x 26,306 / 6

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(a) 1/6
(b) 12 / 6
(c) 26,306 / 6
(d) 12 x 26,306 / 6 = 52,612

Odds of 1's, 2's etc. on one roll for one die:
1/6

Odds of 1's, 2's etc. on one roll for 12 dice:
1/6 * 12 = 12/6

Expected number of 1's, 2's etc. for 26,306 rolls of 12 dice:
12/6 * 26306

Summarizing Labby's results

The table below shows the observed and expected counts from Labby's experiment.

Outcome	Observed	Expected
1	53,222	52,612
2	52,118	52,612
3	52,465	52,612
4	52,338	52,612
5	52,244	52,612
6	53,285	52,612
Total	315,672	315,672

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We can answer this question with a Chi-squared GOF test

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H_0: There is no inconsistency between the observed and the expected counts.
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(There is a bias in which side comes up on the roll of a die.)

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- Large deviations from what would be expected based on sampling variation (chance) alone provide strong evidence for the alternative hypothesis.
- This is called a *goodness of fit* test since we're evaluating how well the observed data fit the expected distribution.

Chi-square statistic

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where k = total number of cells

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3	52,465	52,612	$\frac{(52{,}465-52{,}612)^2}{52{,}612} = 0.41$
4	52,338	52,612	$\frac{(52{,}338-52{,}612)^2}{52{,}612} = 1.43$
5	52,244	52,612	$\frac{(52{,}244-52{,}612)^2}{52{,}612} = 2.57$
6	53,285	52,612	$\frac{(53{,}285-52{,}612)^2}{52{,}612} = 8.61$
Total	315,672	315,672	24.73

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Remember

So far we've seen three other continuous distributions:

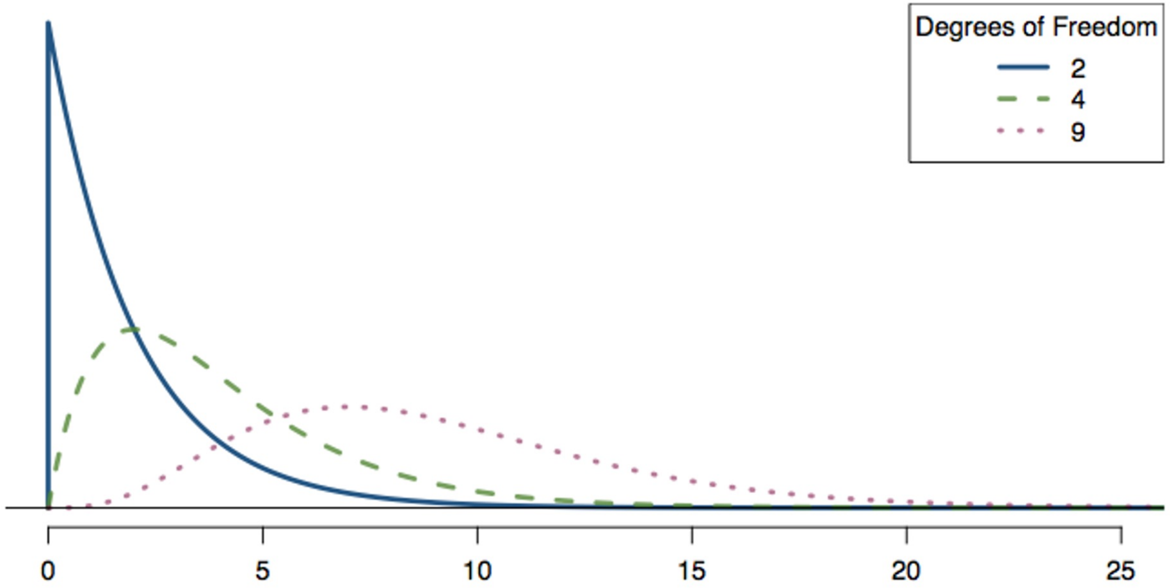
→ Z (normal) distribution: unimodal and symmetric with two parameters: mean and standard deviation

→ T distribution: unimodal and symmetric with one parameter: degrees of freedom

→ F distribution: unimodal and right skewed with two parameters: degrees of freedom or numerator (between group variance) and denominator (within group variance)

Practice

Which of the following is false?



As the df increases,

(a) the center of the χ^2 distribution increases as well

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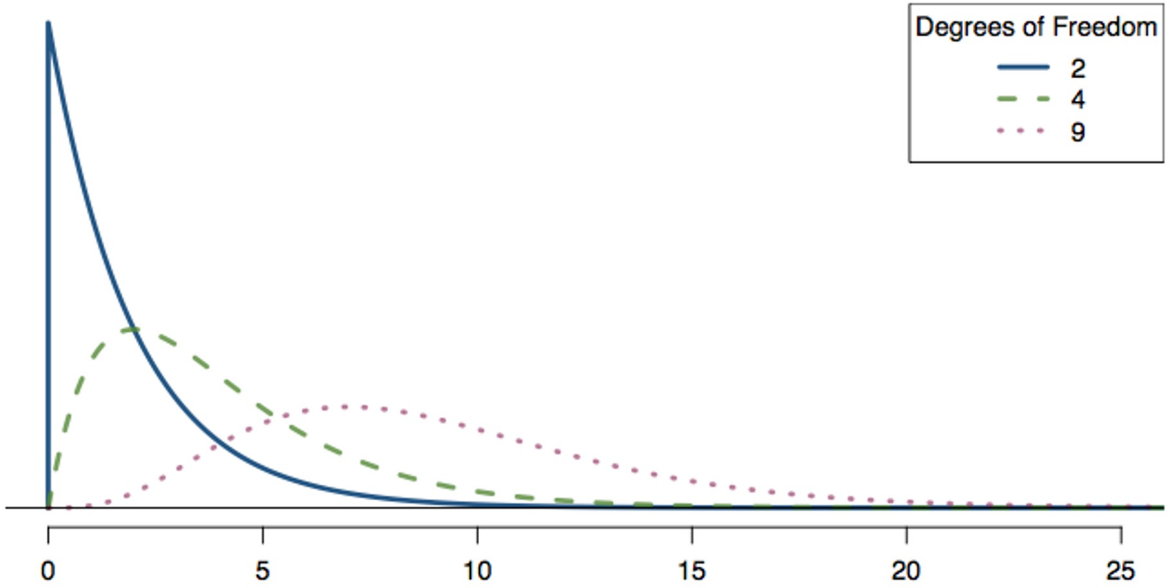

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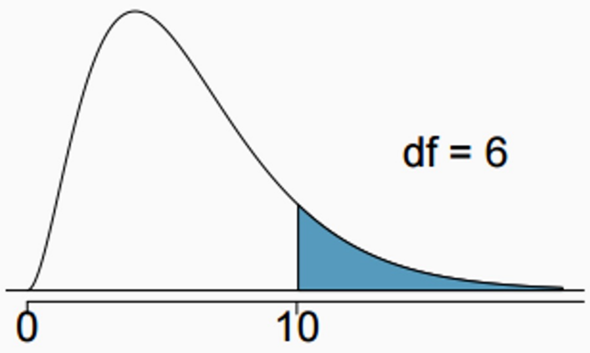
- For this we can use technology, or a *chi-square probability table*.

Finding areas under the chi-square curve

Estimate the shaded area under the chi-square curve with *df = 6*.



Upper tail	0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df 1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83
2	2.41	3.22	4.61	5.99	7.82	9.21	10.60	13.82
3	3.66	4.64	6.25	7.81	9.84	11.34	12.84	16.27
4	4.88	5.99	7.78	9.49	11.67	13.28	14.86	18.47
5	6.06	7.29	9.24	11.07	13.39	15.09	16.75	20.52
6	7.23	8.56	10.64	12.59	15.03	16.81	18.55	22.46
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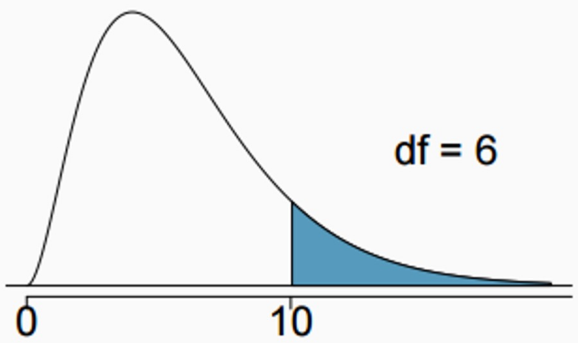

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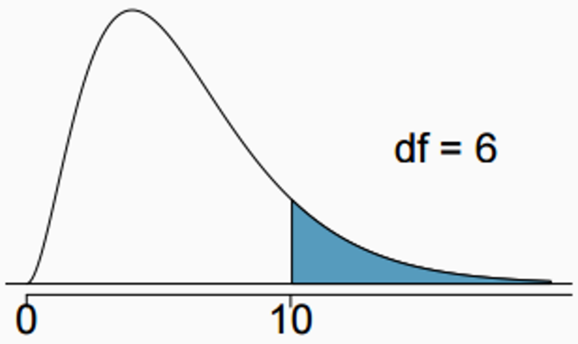

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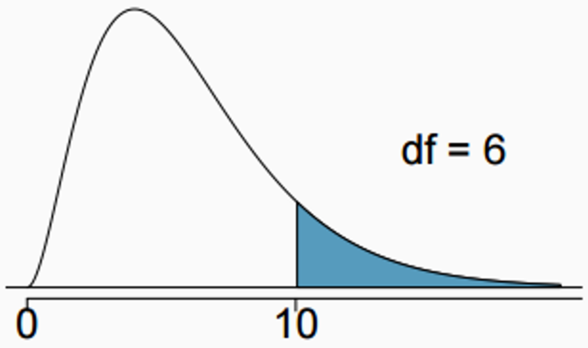

$P(\chi^2_{df=6} > 10)$

is between 0.1 and 0.2

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- We had calculated a test statistic of $\chi^2 = 24.67$.
- All we need is the df and we can calculate the tail area (the p-value) and make a decision on the hypotheses.

Degrees of freedom for a goodness of fit test

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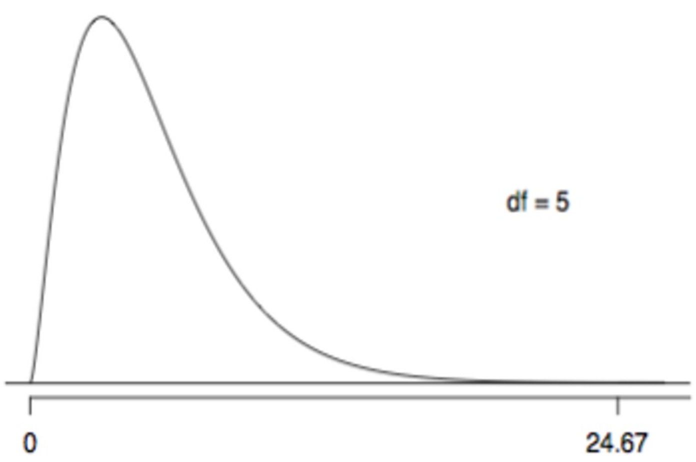
- For dice outcomes, k = 6, therefore

$$df = 6 - 1 = 5$$

Finding a p-value for a chi-square test

The *p-value* for a chi-square test is defined as the *tail area above the calculated test statistic*.



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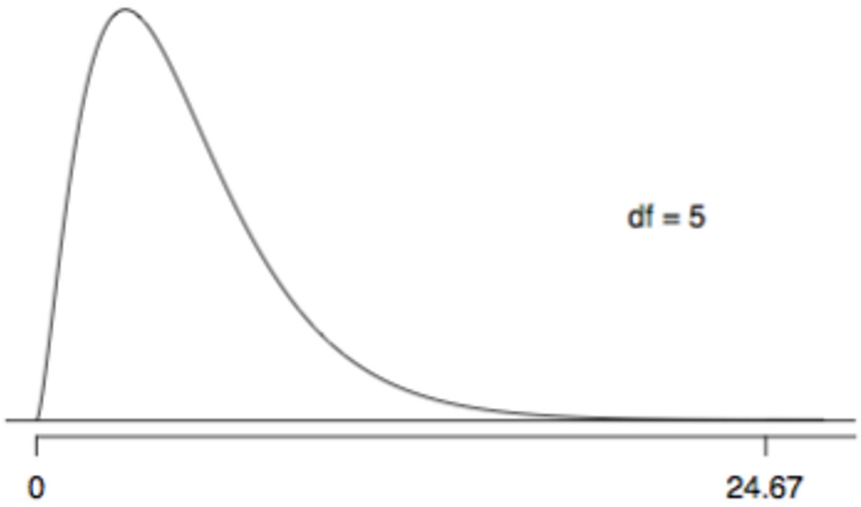

p-value = $P(\chi^2_{df=5} > 24.67)$
is less than 0.001

Conclusion of the hypothesis test

We calculated a p-value less than 0.001. At 5% significance level, what is the conclusion of the hypothesis test?

(a) Reject H_0, the data provide convincing evidence that the dice are fair.
(b) Reject H_0, the data provide convincing evidence that the dice are biased.
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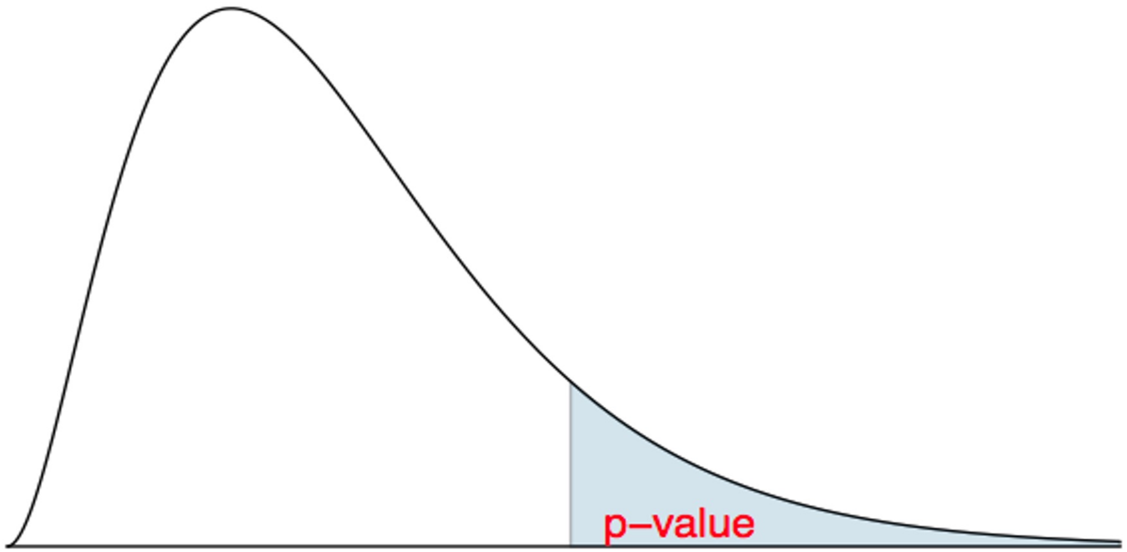
Turns out...

- The 1-6 axis is consistently shorter than the other two (2-5 and 3-4), thereby supporting the hypothesis that the faces with one and six pips are larger than the other faces.
- Pearson's claim that 5s and 6s appear more often due to the carved-out pips is not supported by these data.
- Dice used in casinos have flush faces, where the pips are filled in with a plastic of the same density as the surrounding material and are precisely balanced.

Recap: p-value for a chi-square test

- The p-value for a chi-square test is defined as the tail area *above* the calculated test statistic.

- This is because the test statistic is always positive, and a higher test statistic means a stronger deviation from the null hypothesis.



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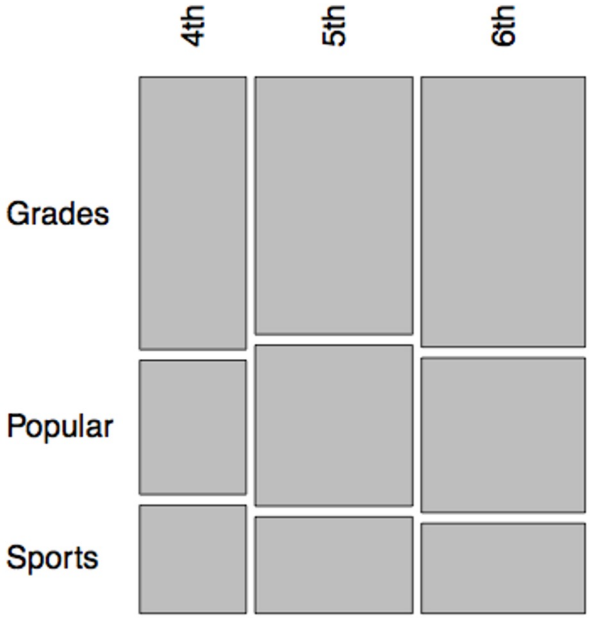
Failing to check conditions may unintentionally affect the test's error rates.

Chi-Square Test of Independence

Motivation: Popular kids

In the dataset `popular`, students in grades 4-6 were asked whether good grades, athletic ability, or popularity was most important to them. A two-way table separating the students by grade and by choice of most important factor is shown below. Do these data provide evidence to suggest that goals vary by grade?

	Grades	Popular	Sports
4^{th}	63	31	25
5^{th}	88	55	33
6^{th}	96	55	32



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- The test statistic is calculated as

$$\chi^2_{df} = \sum_{i=1}^{k} \frac{(O - E)^2}{E} \qquad\qquad df = (R - 1) * (C - 1)$$

where k is the number of cells, R is the number of rows, and C is the number of columns in our contingency table, O is observed counts, and E is expected counts

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Note: *we calculate df differently for one-way and two-way tables.*

Expected counts in two-way tables

Observed

Factor / Grade	Grades	Popular	Sports	Total
4th	63	31	25	119
5th	88	55	33	176
6th	96	55	32	183
Total	247	141	90	478

Expected counts in two-way tables

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$$(R1, C1) = \frac{119\ * 247}{478} = 61$$

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6th	?	?	?	183
Total	247	141	90	478

Expected counts in two-way tables

Observed

Grade \ Factor	Grades	Popular	Sports	Total
4^{th}	63	31	25	119
5^{th}	88	55	33	176
6^{th}	96	55	32	183
Total	247	141	90	478

$$E = \frac{row\ total\ * column\ total}{table\ total}$$

$$(R2, C1) = \frac{176\ * 247}{478} = 91$$

Fill in the rest of the table

Expected

Grade \ Factor	Grades	Popular	Sports	Total
4^{th}	61	?	?	119
5^{th}	91	?	?	176
6^{th}	?	?	?	183
Total	247	141	90	478

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Calculating the test statistic in two-way tables

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$$\chi^2_{df} = \sum_{i=1}^{k} \frac{(O - E)^2}{E} \qquad df = (R - 1) * (C - 1)$$

What is the Chi-squared statistic?

Calculating the test statistic in two-way tables

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Grade \ Factor	Grades	Popular	Sports	Total
4th	63	31	25	119
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6th	96	55	32	183
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$$\chi^2_{df} = \sum_{i=1}^{k} \frac{(O - E)^2}{E} \qquad df = (R - 1) * (C - 1)$$

$$\chi^2_{df} = \sum_{i=1}^{9} \frac{(O - E)^2}{E}$$

$$= \frac{(63-61)^2}{61} + \frac{(31-35)^2}{35} + \frac{(25-23)^2}{23} + \frac{(88-91)^2}{91} + \frac{(55-52)^2}{52} + \frac{(33-33)^2}{33} + \frac{(96-95)^2}{95} + \frac{(55-54)^2}{54} + \frac{(32-34)^2}{34}$$

$$= 1.3121$$

Calculating the test statistic in two-way tables

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$$\chi^2_{df} = \sum_{i=1}^{k} \frac{(O - E)^2}{E} \qquad df = (R - 1) * (C - 1)$$

$$\chi^2_{df} = 1.3121$$

What are the degrees of freedom?

Calculating the test statistic in two-way tables

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$$\chi^2_{df} = \sum_{i=1}^{k} \frac{(O - E)^2}{E} \qquad df = (R - 1) * (C - 1)$$

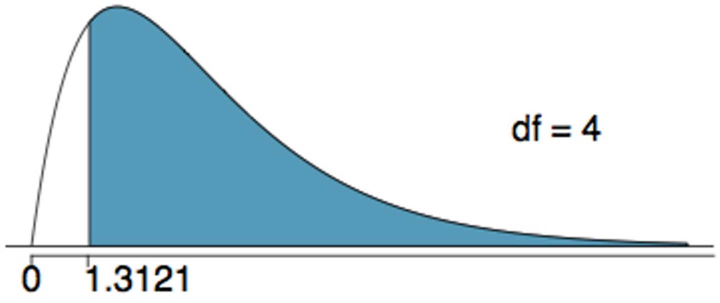
$\chi^2_{df} = 1.3121$

$df = (3 - 1) * (3 - 1) = 4$

Calculating the p-value

Which of the following is the correct p-value for this hypothesis test?

$\chi^2_{df} = 1.3121$ df = 4



(a) more than 0.3
(b) between 0.3 and 0.2
(c) between 0.2 and 0.1
(d) between 0.1 and 0.05
(e) less than 0.001

Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83
	2	2.41	3.22	4.61	5.99	7.82	9.21	10.60	13.82
	3	3.66	4.64	6.25	7.81	9.84	11.34	12.84	16.27
	4	4.88	5.99	7.78	9.49	11.67	13.28	14.86	18.47
	5	6.06	7.29	9.24	11.07	13.39	15.09	16.75	20.52

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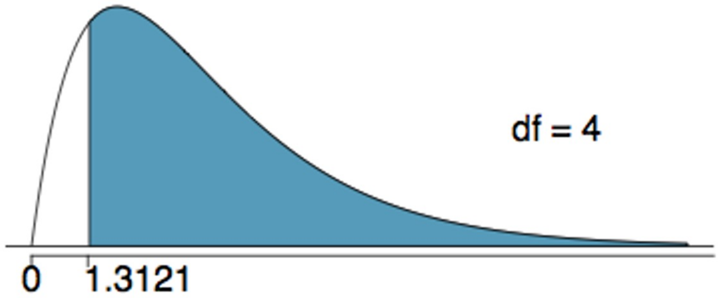

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Conclusion

Do these data provide evidence to suggest that goals vary by grade?

$$p - value > 0.3$$

H_0: Grade and goals are independent. Goals do not vary by grade.

H_A: Grade and goals are dependent. Goals vary by grade.

Conclusion

Do these data provide evidence to suggest that goals vary by grade?

$$p - value > 0.3, \alpha = 0.05$$

H_0: Grade and goals are independent. Goals do not vary by grade.
H_A: Grade and goals are dependent. Goals vary by grade.

Since the p-value is large, we fail to reject H_0. The data do not provide convincing evidence that grade and goals are dependent. It doesn't appear that goals vary by grade.

Practice

2009 Iran Election

There was lots of talk of election fraud in the 2009 Iran election. We'll compare the data from a poll conducted before the election (observed data) to the reported votes in the election to see if the two follow the same distribution.

Candidate	Observed # of voters in poll	Reported % of votes in election
(1) Ahmedinajad	338	63.29%
(2) Mousavi	136	34.10%
(3) Minor candidates	30	2.61%
Total	504	100%

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Total	504	100%
	observed	*expected distribution*

Hypotheses

What are the hypotheses for testing if the distributions of reported and polled votes are different?

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What are the hypotheses for testing if the distributions of reported and polled votes are different?

H_0: The observed counts from the poll follow the same distribution as the reported votes.

H_A: The observed counts from the poll do not follow the same distribution as the reported votes.

Calculation of the test statistic

Candidate	Observed # of voters in poll	Reported % of votes in election	Expected # of votes in poll
(1) Ahmedinajad	338	63.29%	$504 \times 0.6329 = 319$
(2) Mousavi	136	34.10%	$504 \times 0.3410 = 172$
(3) Minor candidates	30	2.61%	$504 \times 0.0261 = 13$
Total	504	100%	504

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Total	504	100%	504

$$\frac{(O_1 - E_1)^2}{E_1} = \frac{(338 - 319)^2}{319} = 1.13$$

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Total	504	100%	504

$$\frac{(O_1 - E_1)^2}{E_1} = \frac{(338 - 319)^2}{319} = 1.13$$

$$\frac{(O_2 - E_2)^2}{E_2} = \frac{(136 - 172)^2}{172} = 7.53$$

Calculation of the test statistic

Candidate	Observed # of voters in poll	Reported % of votes in election	Expected # of votes in poll
(1) Ahmedinajad	338	63.29%	504 × 0.6329 = 319
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Total	504	100%	504

$$\frac{(O_1 - E_1)^2}{E_1} = \frac{(338 - 319)^2}{319} = 1.13$$

$$\frac{(O_2 - E_2)^2}{E_2} = \frac{(136 - 172)^2}{172} = 7.53$$

$$\frac{(O_2 - E_2)^2}{E_2} = \frac{(30 - 13)^2}{13} = 22.23$$

Calculation of the test statistic

Candidate	Observed # of voters in poll	Reported % of votes in election	Expected # of votes in poll
(1) Ahmedinajad	338	63.29%	504 × 0.6329 = 319
(2) Mousavi	136	34.10%	504 × 0.3410 = 172
(3) Minor candidates	30	2.61%	504 × 0.0261 = 13
Total	504	100%	504

$$\frac{(O_1 - E_1)^2}{E_1} = \frac{(338 - 319)^2}{319} = 1.13$$

$$\frac{(O_2 - E_2)^2}{E_2} = \frac{(136 - 172)^2}{172} = 7.53$$

$$\frac{(O_2 - E_2)^2}{E_2} = \frac{(30 - 13)^2}{13} = 22.23$$

$$\chi^2_{df=3-1=2} = 30.89$$

Conclusion

Based on these calculations what is the conclusion of the hypothesis test?

(a) p-value is low, H_0 is rejected. The observed counts from the poll do <u>not</u> follow the same distribution as the reported votes.
(b) p-value is high, H_0 is not rejected. The observed counts from the poll follow the same distribution as the reported votes.
(c) p-value is low, H_0 is rejected. The observed counts from the poll follow the same distribution as the reported votes
(d) p-value is low, H_0 is not rejected. The observed counts from the poll do *not* follow the same distribution as the reported votes.

Conclusion

Based on these calculations what is the conclusion of the hypothesis test?

(a) *p-value is low, H_0 is rejected. The observed counts from the poll do not follow the same distribution as the reported votes.*

(b) p-value is high, H_0 is not rejected. The observed counts from the poll follow the same distribution as the reported votes.

(c) p-value is low, H_0 is rejected. The observed counts from the poll follow the same distribution as the reported votes

(d) p-value is low, H_0 is not rejected. The observed counts from the poll do *not* follow the same distribution as the reported votes.