Intro to Coding with Python– Recursion Pt. 1

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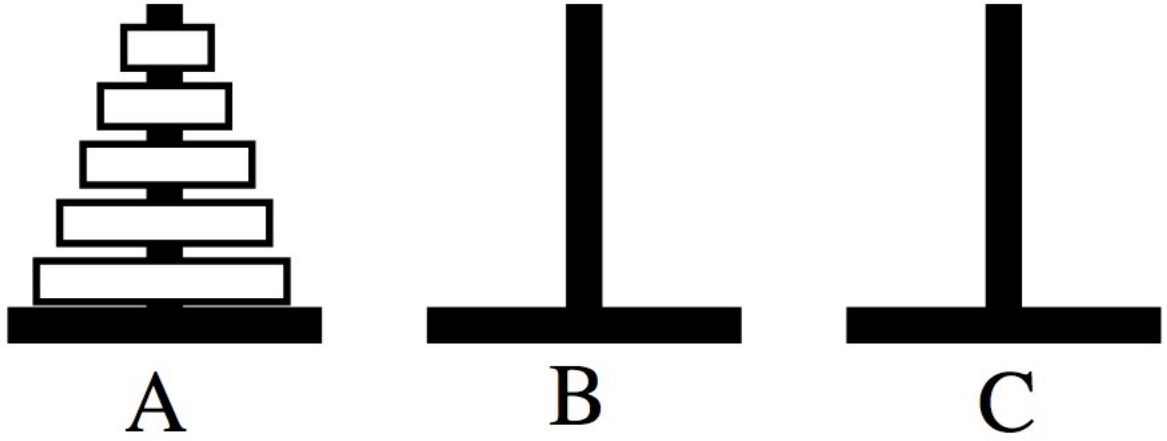
Slides based off slides courtesy of Jordan Crouser (https://jcrouser.github.io/)

Plan for Today

- Motivating example: Towers of Hanoi
- Tough problems, simple solutions

Towers of Hanoi



- Move the tower from A to C
- You can only move **one** disk at a time
- You can only move a disk to a pole where it will be the **smallest** (i.e. you can't put a disk on top of a larger one)
- You can only remove the **smallest** disk from a pole (i.e. you can't lift up the stack to get a larger disk from below)

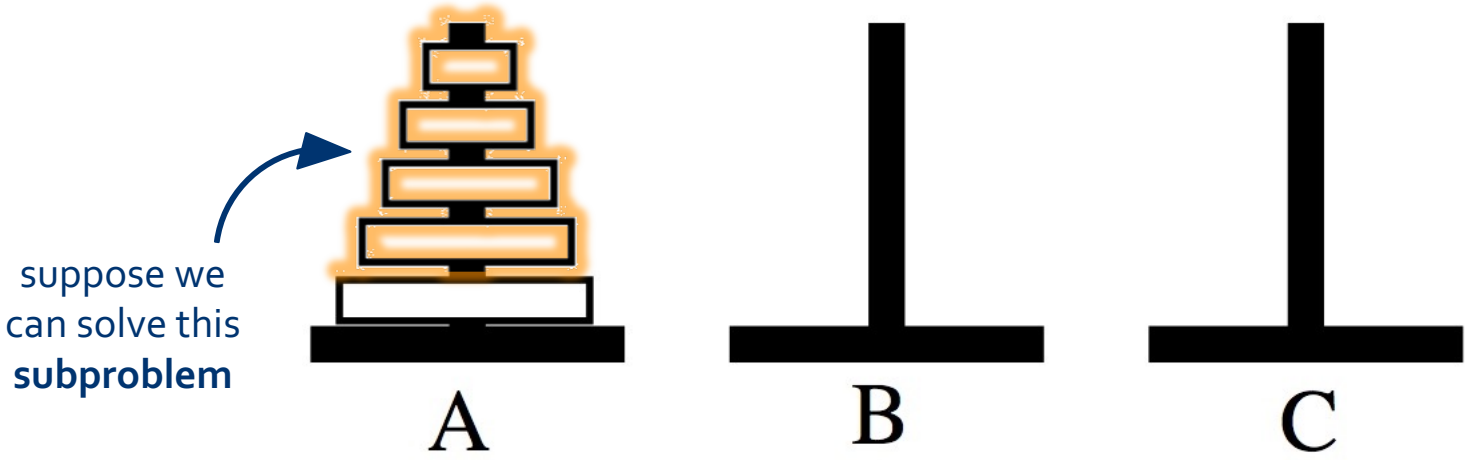
Discussion

Did you solve it?

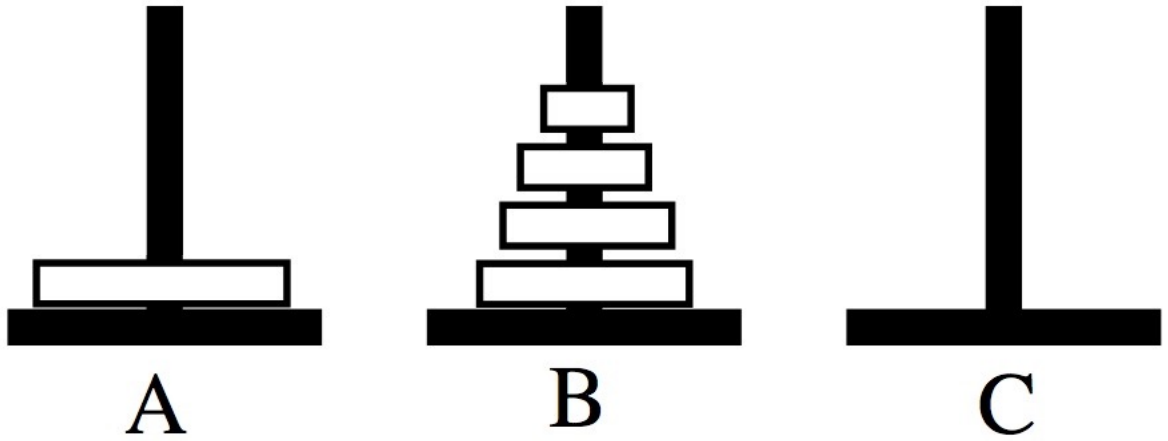
Notice any **patterns**?

Recursive Towers

What if we re-frame the problem this way?

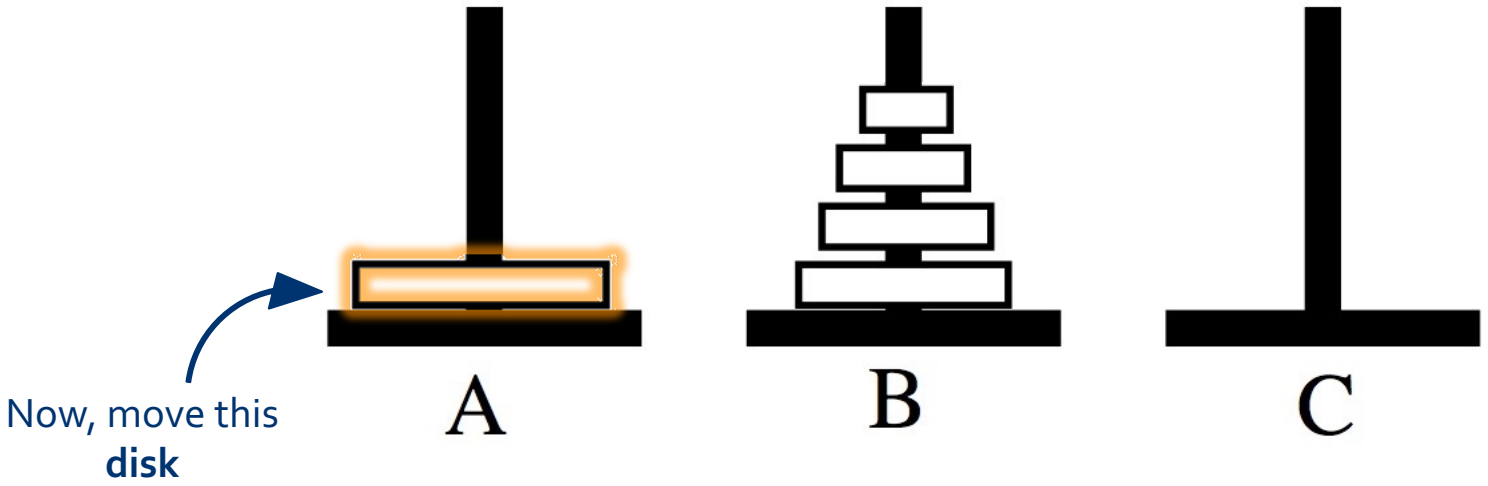


Recursive Towers

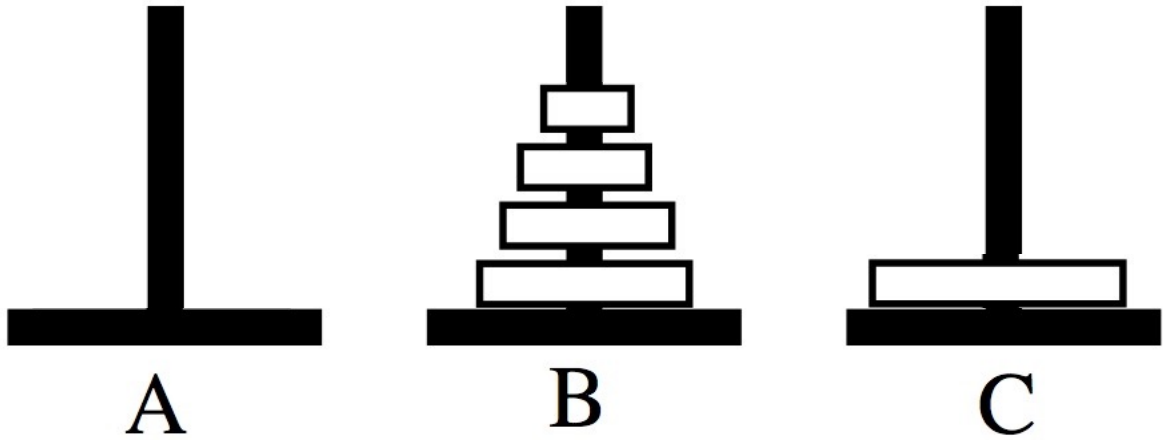


Solved!

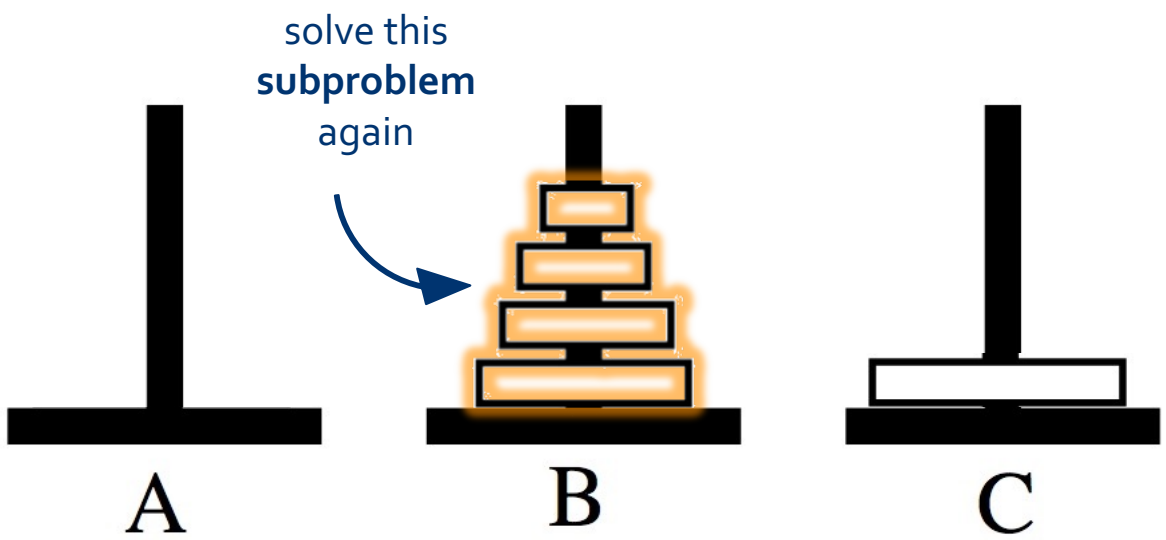
Recursive Towers



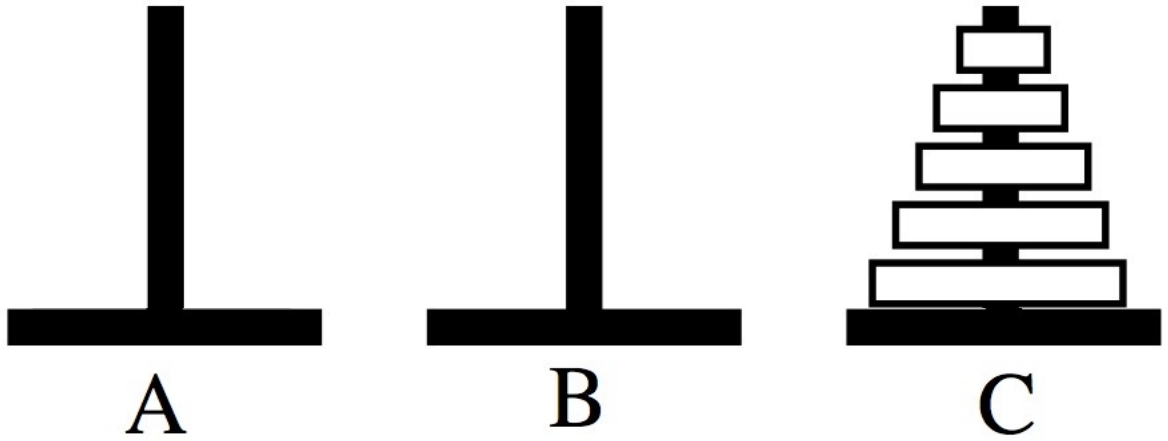
Recursive Towers



Recursive Towers

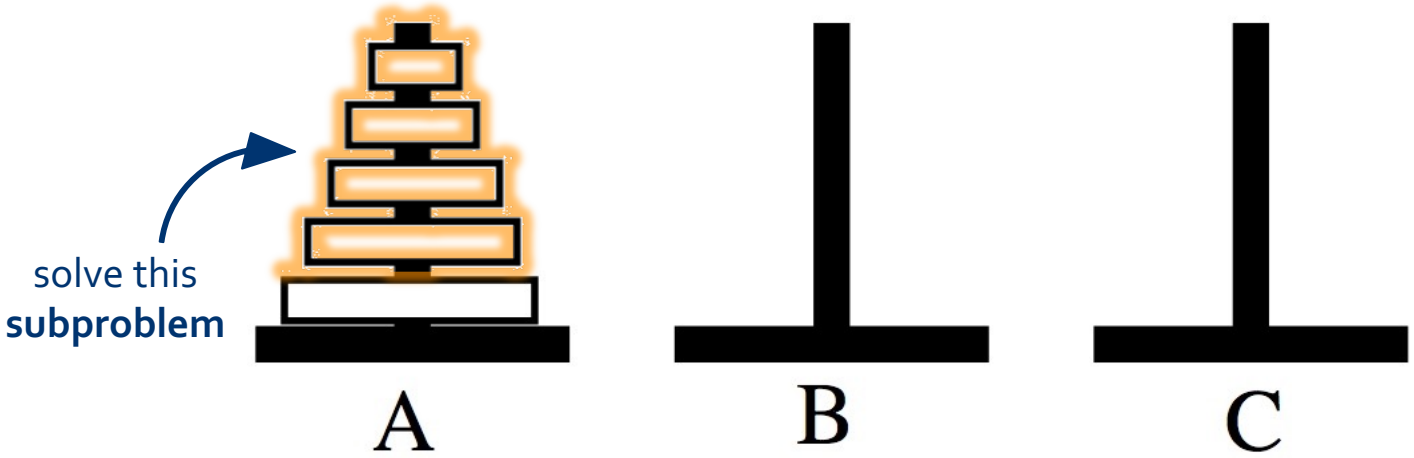


Recursive Towers

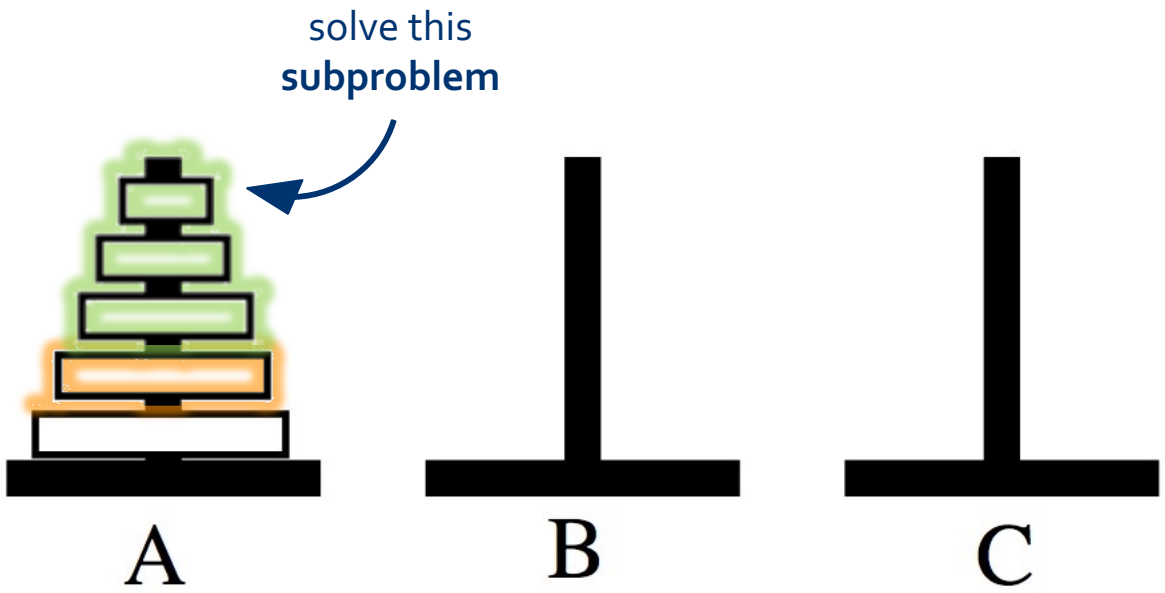


Recursive Towers

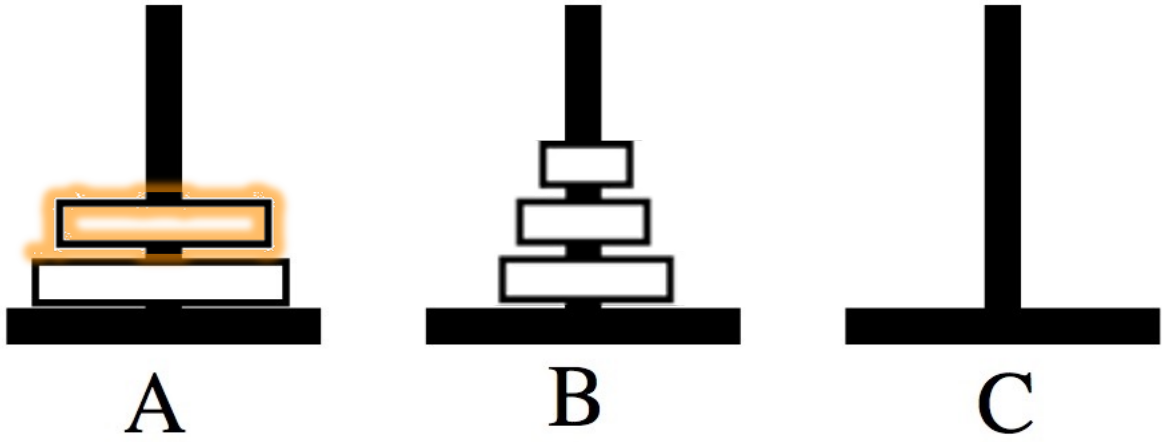
Back to this subproblem. We can think about it the same way



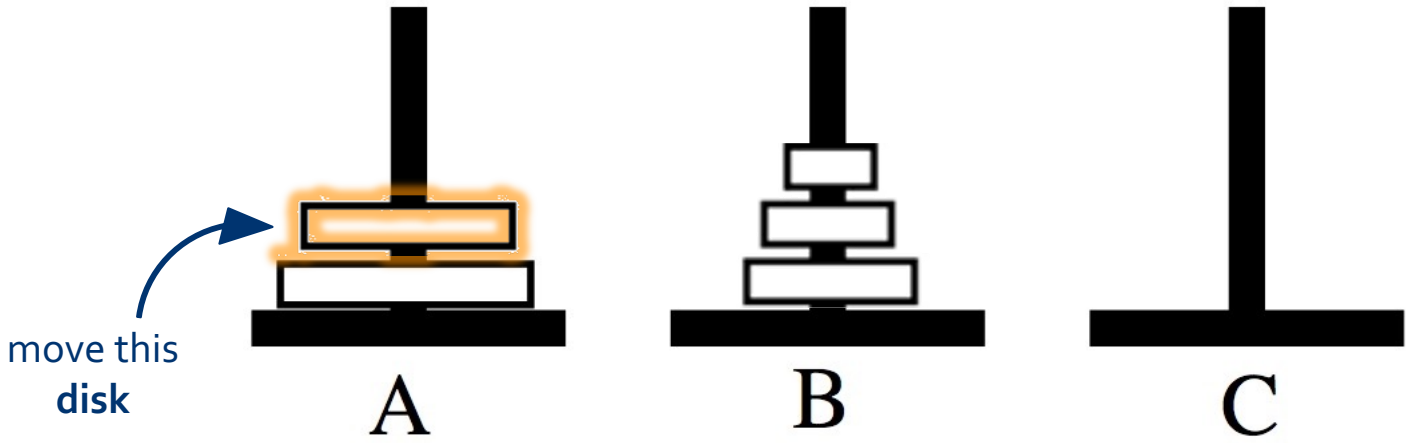
Recursive Towers



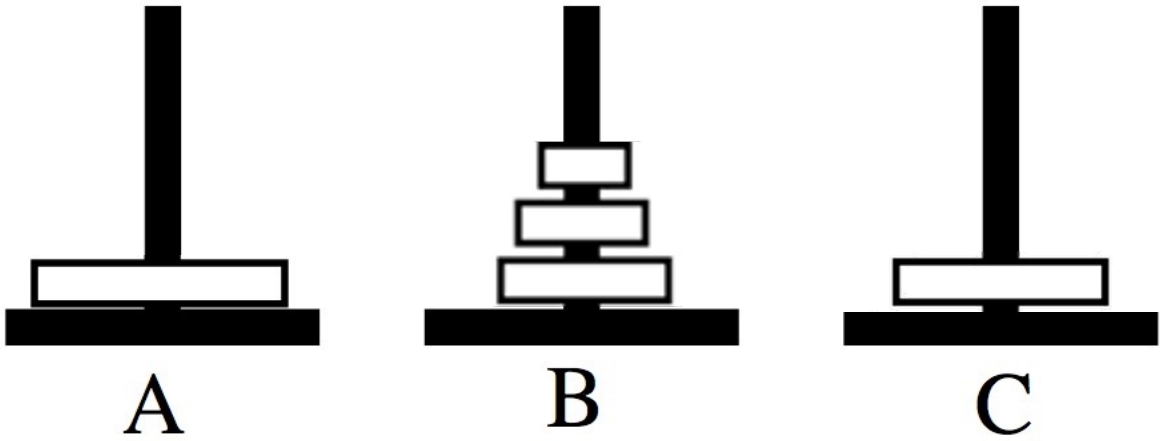
Recursive Towers



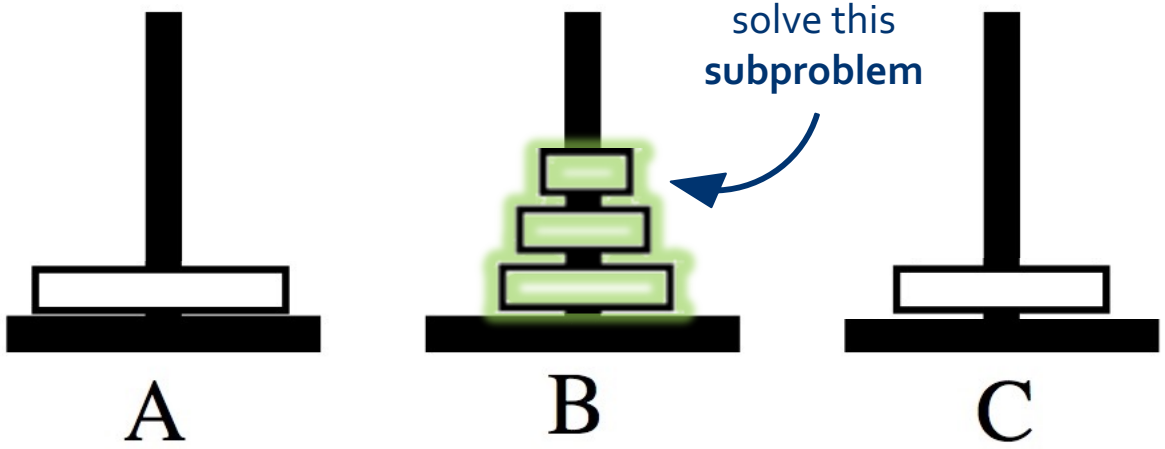
Recursive Towers



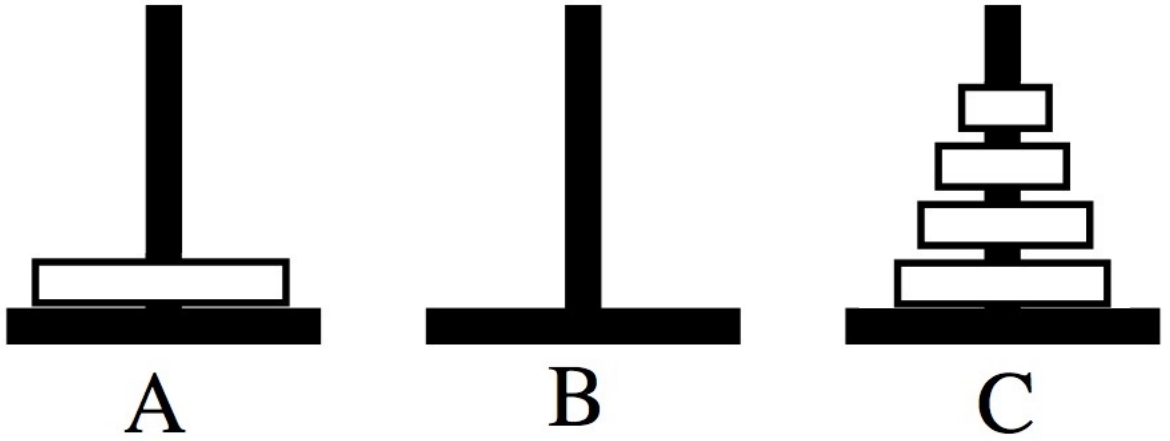
Recursive Towers



Recursive Towers



Recursive Towers



...and so on!

Discussion

Try the towers again. Start with one disk, then 2, then 3, …

How many **moves** does it take to solve each version?

Algorithmic analysis

nDisks	nMoves
1	1
2	3
3	7
4	15
5	31
6	64
7	127

Notice any **patterns**?

$$nMoves = 2^{nDisks} - 1$$

Basic structure of a recursive algorithm

- **A base case**: what to do in the simplest possible case (i.e. when you have a single disk)
- **A recursive step**: break the original problem into one or more smaller problems, and solve that (saving the intermediate result)

Demo: Towers of Hanoi in Python